PARAMETRIZATIONS OF THE SEESAW or CAN THE SEESAW BE TESTED?

SACHA DAVIDSON *

Dept of Physics, University of Durham, Durham, DH1 3LE, England
E-mail: sacha.davidson@durham.ac.uk

This proceedings contains a review, followed by a more speculative discussion. I review different coordinate choices on the 21-dimensional parameter space of the seesaw, and which of these 21 quantities are observable. In MSUGRA, there is a 1-1 correspondance between the parameters, and the interactions of light (s)particles. However, not all of the 21 can be extracted from data, so the answer to the title question is “no”. How to parametrise the remaining unknowns is confusing—different choices seem to give contradictory results (for instance, to the question “does the Baryon Asymmetry depend on the CHOOZ angle?”). I speculate on possible resolutions of the puzzle.

1. Introduction

The seesaw mechanism [1] is a theoretically elegant way to get the small neutrino masses we observe. It predicts that the light neutrino masses are majorana, which could be verified in neutrinoless double β decay experiments. In the absence of Supersymmetry, it predicts that lepton flavour violation (LFV), and CP violation are suppressed by powers of the neutrino mass, making the rates very low outside the neutrino sector. On the other hand, if spartners were discovered, for instance at the LHC, observable CP and flavour violation can be imprinted by the seesaw into the slepton mass matrices. Experimentally verifying these predictions would increase our confidence in the seesaw. Measuring something different—for instance majorana masses, no SUSY, and large neutrino magnetic moments—would indicate that there is other new physics, or more new physics in the lepton sector than just the seesaw (see *e.g.* Smirnov, in this volume). The aim

*work supported by a PPARC Advanced Fellowship

here is to ask if we can *test* the seesaw, or as discussed below, a particular implementation of the seesaw mechanism.

This proceedings is written from a bottom-up phenomenological perspective. I want to make as few assumptions as possible about the theory at scales above m_W, so I assume the particle content is the Standard Model (SM), or the MSSM with universal soft masses, plus three ν_R, and allow all possible renormalisable interactions. This gives the Lagrangian (in the SM case)

$$L = Y_e \bar{e}_R H_d \cdot \ell_L + Y_\nu \bar{\nu}_R H_u \cdot \ell_L + \frac{M}{2} \bar{\nu^c}_R \nu_R + h.c. \tag{1}$$

where ℓ are the lepton doublets, $\bar{\nu^c}_R = (C\nu_R^*)^\dagger \gamma_0$, and generation indices are suppressed. The index order on the Yukawa matrices is right-left.

To test this implementation of the seesaw mechanism, we need to

(1) extract the unknown parameters of eqn (1) from data
(2) predict an additional observable calculated from those parameters
(3) verify the prediction

These proceedings discuss the first step. If it could be accomplished successfully, we could calculate the baryon asymmetry produced in various leptogenesis[2,3,4] mechanisms (see Hambye and Raidal in this volume), which would be a fabulous cross-check of particle physics and cosmology.

There are many other versions of the seesaw (2 ν_R, type II with scalar triplets, with extra singlets...), which are motivated from various theoretical perspectives (see T Hambye in this volume). The model used here contains three ν_R because there are three generations, and only three ν_R because it is useful to know how well the simple model works before adding complications.

I want to test the seesaw *mechanism*, rather than a particular model, so GUT models, textures, and theoretical considerations of "naturalness" are avoided (insofar as possible). The seesaw mechanism can accomodate *any* neutrino masses and mixing angles (And almost any sneutrino mass matrix[6]). Particular models may prefer certain ranges for observables, so data can provide hints about the theory that gives the Lagrangian of eqn (1). This is discussed elsewhere in this volume (G Ross and P Ramond). However, if these theoretical expectations are not fulfilled, it is difficult to know if the model was wrong, or if there is more new physics in addition to the seesaw mechanism.

2. Parametrisations

Twenty-one parameters are required[5] to fully determine the Lagrangian of eqn (1). Three of the possible ways these can be chosen are discussed here.

The usual **"top-down"** description of the theory is as follows. At energy scales $\Lambda \gtrsim M$, where the ν_R are propagating degrees of freedom, one can always choose the ν_R basis where the mass matrix M is diagonal, with positive and real eigenvalues: $M = D_M$. Similarly, one can choose the ℓ basis such that the charged lepton Yukawa Y_e is diagonal on its LH indices: $Y_e^\dagger Y_e = D_{Y_e}^2$. The remaining neutrino Yukawa matrix Y_ν is an arbitrary complex matrix, from which three phases can be removed by phase redefinitions on the ℓ_i. It is therefore described by 9 moduli and 6 phases, giving in total 21 real parameters for the seesaw. See [5] for a more elegant counting, in particular of the phases.

To relate various parametrisations of the seesaw, it is useful to diagonalise Y_ν, which can be done with independent unitary transformations on the left and right:

$$Y_\nu = V_R^\dagger D_{Y_\nu} V_L \tag{2}$$

So in the top-down approach, the lepton sector can be described by the nine eigenvalues of D_M, D_{Y_ν} and D_{Y_e}, and the six angles and six phases of V_L and V_R. Notice that in this parametrisation, the inputs are masses and coupling constants of the propagating particles at energies Λ, so it makes "physical" sense.

The effective mass matrix m of the light neutrinos can be calculated, in the D_{Y_e} basis (charged lepton mass eigenstate basis):

$$m \equiv \kappa \langle H_u \rangle^2 = Y_\nu^T D_M^{-1} Y_\nu \langle H_u \rangle^2 = V_L^T D_{Y_\nu} V_R^* D_M^{-1} V_R^\dagger D_{Y_\nu} V_L \langle H_u \rangle^2 \tag{3}$$

κ is introduced to avoid the Higgs vev $\langle H_u \rangle$ cluttering up formulae. The leptonic mixing matrix U is extracted by diagonalising κ:

$$\kappa = U^* D_\kappa U^\dagger \tag{4}$$

where $D_\kappa = diag\{\kappa_1, \kappa_2, \kappa_3\}$, and U is parametrised as

$$U = \hat{U} \cdot \mathrm{diag}(1, e^{i\alpha}, e^{i\beta}) \quad . \tag{5}$$

α and β are "Majorana" phases, and $\hat{U}$ has the form of the CKM matrix

$$\hat{U} = \begin{bmatrix} c_{13}c_{12} & c_{13}s_{12} & s_{13}e^{-i\delta} \\ -c_{23}s_{12} - s_{23}s_{13}c_{12}e^{i\delta} & c_{23}c_{12} - s_{23}s_{13}s_{12}e^{i\delta} & s_{23}c_{13} \\ s_{23}s_{12} - c_{23}s_{13}c_{12}e^{i\delta} & -s_{23}c_{12} - c_{23}s_{13}s_{12}e^{i\delta} & c_{23}c_{13} \end{bmatrix} \quad . \tag{6}$$

Alternatively, the (type I) seesaw Lagrangian of eqn (1) can be described with inputs from the left-handed sector [6]. This is refered to as a **"bottom-up"** parametrisation, because the left-handed (SU(2) doublet) particles have masses $\lesssim$ the weak scale. D_{Y_e}, U and D_κ, can be taken as a subset of the inputs. To identify the remainder, imagine sitting in the ℓ basis where κ is diagonal, so as to emphasize the parallel between this parametrisation and the previous one (this is similar to the ν_R basis being chosen to diagonalise M). If one knows $Y_\nu^\dagger Y_\nu \equiv W_L D_{Y_\nu}^2 W_L^\dagger$ in the D_κ basis, then the ν_R masses and mixing angles can be calculated:

$$M^{-1} = D_Y^{-1} W_L^* D_\kappa W_L^\dagger D_Y^{-1} = V_R^* D_M^{-1} V_R^\dagger \tag{7}$$

In this parametrisation, there are three possible basis choices for the ℓ vector space: the charged lepton mass eigenstate basis (D_{Y_e}), the neutrino mass eigenstate basis (D_κ), and the basis where the Y_ν is diagonal. The first two choices are physical, that is, U rotates between these two bases. D_{Y_e},D_κ and U contain the 12 possibly measurable parameters of the SM seesaw. The remaining 9 parameters can be taken to be D_{Y_ν} and V_L (or $W = V_L U$). In SUSY one can hope to extract these parameters from the slepton mass matrix.

The **Casas-Ibarra** [7] parametrisation is very convenient for calculations. It uses D_M, D_κ and D_{Y_e} as inputs, and the transformations U and R, which go between the bases where these matrices are diagonal. U is the usual leptonic mixing matrix. The matrix $R = D_M^{-1/2} Y_\nu D_\kappa^{-1/2}$, is a complex *orthogonal* matrix, which transforms between the D_M and D_κ bases. (Since M and κ are respectively in the RH and LH neutrino vector spaces, it is unsurprising that the transformation matrix is not unitary.) R can be written as $R = \mathrm{diag}\{\pm 1, \pm 1, \pm 1\}\hat{R}$ where the ± 1 are related to the CP parities of the N_i, and $\hat{R}$ is an orthogonal matrix with complex angles:

$$\hat{R} = \begin{bmatrix} c_{13}c_{12} & c_{13}s_{12} & s_{13} \\ -c_{23}s_{12} - s_{23}s_{13}c_{12} & c_{23}c_{12} - s_{23}s_{13}s_{12} & s_{23}c_{13} \\ s_{23}s_{12} - c_{23}s_{13}c_{12} & -s_{23}c_{12} - c_{23}s_{13}s_{12} & c_{23}c_{13} \end{bmatrix} \quad . \tag{8}$$

The aim of this proceedings is to reconstruct the RH seesaw parameters from the LH ones, many of which are accessible at low energy. However,as discussed in the following section, reconstruction is impossible. We can at best try to establish relations between observables, which turns out to be quite confusing. R will be helpful in discussing these puzzles.

In summary, the lepton sector of the SM + seesaw can be parametrised with D_{Y_e}, the real eigenvalues of two more matrices, and the transforma-

tions among the bases where the matrices are diagonal. The matrices-to-be-diagonalised can be chosen in various ways:

(1) " top-down"—input the ν_R sector: D_M, $D_{Y_\nu Y_\nu^\dagger}$, and V_R and V_L.
(2) " bottom-up"—input the ν_L sector: D_κ, $D_{Y_\nu^\dagger Y_\nu}$, and V_L and U.
(3) "intermediate"—the Casas-Ibarra parametrization: D_M, D_κ, and U and a complex orthogonal matrix R.

3. (Supersymmetric) reconstruction?

If the matrices D_κ, D_{Y_e}, $D_{Y_\nu^\dagger Y_\nu}$, V_L and U were known, it would be possible to reconstruct the masses and mixing angles of the ν_R. Can the elements of these matrices be determined [6]?

We know the masses of the charged leptons, so we know D_{Y_e} (modulo $\tan\beta$ in SUSY models).

We know two mass differences in the neutrino sector. If the light neutrinos are degenerate, measuring the overall scale of their masses is possible and would determine D_κ. However, if the mass pattern is hierarchical or inverse hierarchical, we would know only κ_3 and κ_2. See the contribution of K Heeger, for present and future accuracy on D_κ, and U.

In the mixing matrix U, we currently know two angles. We hope to measure the third, and also the "Dirac" phase δ. But the "majorana" phases appear only in slow lepton number changing processes, so at the moment do not seem experimentally accessible [8].

The remaining parameters to be determined are the eigenvalues of Y_ν, and the matrix V_L. In supersymmetric models Y_ν contributes via loops to the slepton mass matrix. Consider a model, such as gravity-[9] or anomaly-mediated [10] SUSY breaking, where the soft masses are universal at a scale $\Lambda > M_3$. In renormalisation group running between Λ and m_W, the slepton mass matrix will acquire flavour off-diagonal terms, due to loops involving the ν_R (see Masiero in this volume). Using the leading log approximation for the RG running, the sneutrino mass matrix, in the D_{Y_e} basis, is:

$$\left[m_{\tilde\nu}^2\right]_{ij} \simeq (\text{diag part}) - \frac{3m_0^2 + A_0^2}{8\pi^2}(\mathbf{Y}_\nu^\dagger)_{ik}(\mathbf{Y}_\nu)_{kj} \log\frac{\Lambda}{M_k} \tag{9}$$

where m_0 and A_0 are the universal soft parameters at scale Λ.

It is tantalising that the seesaw contribution to flavour violation in the sleptons is potentially observable, and depends on the heavy neutrino masses in a different way than κ. *If* we could determine $\left[m_{\tilde\nu}^2\right]$ exactly (the three masses, three mixing angles, and three phases), and *if* we take seriously the assumption of universal soft masses, then we could reconstruct

the renormalisable interactions of the high-scale seesaw—that is, the ν_R masses and Yukawa couplings—from the mass matrices and mixing angles of weak-scale particles.

Unfortunately, neither of these conditions is likely to be fulfilled. Firstly, not all the parameters of $\left[m_{\tilde{\nu}}^2\right]$ can be measured with the required accuracy. The diagonal elements of the second term of eqn (9) shift $m_{\tilde{\nu}}^2$ by of order y_i^2%, so a large Y_ν eigenvalue ~ 1 could have a measurable effect. However, if Y_ν has a hierarchy similar to the quark Yukawas, the effects of the first and second generation y_i are (undetectably) small.

The flavour-changing elements of eqn (9) could be seen at colliders [11], and induce rare decays, such as $\mu \to e\gamma$ [12]. A very optimistic experimental sensitivity of order $BR(\tau \to \ell\gamma) \sim 10^{-9}$ (the current limit is $\sim 10^{-7}$), could probe $|[V_L]_{3\ell}[V_L]^*_{3\tau}y_3^2| \gtrsim 10^{-(1\div 2)}$. $\mu - e$ flavour violation is more encouraging: there are plans to reach $BR(\mu \to e\gamma) \sim 10^{-13}$, which would be sensitive to $|[V_L]_{3e}[V_L]^*_{3\mu}y_3^2| \gtrsim 10^{-(3\div 4)}$. However, to extract a "measurement" of either of the $|[V_L]_{3\ell}|$ from rare decays would require knowing all the masses and mixing angles for the other SUSY particles contributing to the decay.

For hierarchical Y_ν eigenvalues, eqn (9) implies that the three off-diagonal elements of $\left[m_{\tilde{\nu}}^2\right]$, are determined by two matrix elements of V_L. So one angle of V_L is unknown, and there should be some correlation between $\left[m_{\tilde{\nu}}^2\right]_{\tau\mu}$, $\left[m_{\tilde{\nu}}^2\right]_{\tau e}$,and $\left[m_{\tilde{\nu}}^2\right]_{\mu e}$. Notice, however, that this is a prediction of hierarchical Y_ν. In the bottom-up parametrisation, the slepton mass matrix determines V_L and D_{Y_ν}, rather than the seesaw making predictions for $\left[m_{\tilde{\nu}}^2\right]$.

Now we come to the three phases of V_L. To extract all of these is quite hypothetical; it would require three independent measurements of CP violation in the sleptons. Two possibilities at colliders are charged lepton asymmetries in slepton decays [13], and sneutino-anti-sneutrino oscillations [14]. The slepton phases also contribute to CP violating observables in the leptons, in conjunction with phases from other SUSY particles. This is discussed in this volume by Hisano.

The second objection to extracting seesaw parameters from eqn (9), is that we do not know that soft masses are universal. It is a reasonable assumption in top-down analyses, because we know that flavour violation mediated by sparticles must be suppressed. But I know of no way to distinguish contributions to $\left[m_{\tilde{\nu}}^2\right]$ that come from the RG running with the seesaw, from those that come from non-universal soft masses, threshhold effects, other particles with flavour off-diagonal couplings, etc... So

measuring $\left[m_{\tilde{\nu}}^2\right]$ exactly could be used to set an upper bound on the seesaw contributions (if one makes the reasonable assumption that there are no cancellations among different contributions), but would not determine them.

It is also possible-in-principle to reconstruct the **non-SUSY** seesaw: Broncano *et al.* [15] observed that the 21 parameters can be extracted from the coefficients of dimension 5 and 6 operators in the Standard Model. However, the coefficients of the dimension 6, lepton number conserving operators are suppressed by two powers of the ν_R mass, so are (unobservably) small.

In summary, the parameters of the type I seesaw cannot be extracted from data. This should hardly be surprising—we do not usually expect to reconstruct high-scale theories (*e.g.* which GUT, and how does it break?) from weak-scale observations. So why do we even ask if it is possible in the seesaw? I am aware of two peculiarities, which make the seesaw "reconstructable in principle": the ν_R only have interactions with light particles (via Y_ν), and the effective operators induced at low energy are experimentally accessible (in principle!) for all flavour indices. To see why these features are significant, compare to proton decay— which I assume to be mediated by a "triplet higgsino" dressed with a squark loop. However accurately we mesure every available proton decay channel, we cannot determine the mass and couplings of the triplet higgsino, because we must always sum over squark flavours in the loop, and we only measure proton decay with first generation quarks in the initial state (unlike the three generation ν and $\tilde{\nu}$ mass matrices).

4. Independence, orthogonality and relations when we cannot reconstruct

The 21 parameters of the seesaw cannot be determined from observation, but some sort of partial reconstruction, using the available data, could be possible. This turns out to be much more confusing than one would anticipate. To identify the problem, imagine calculating the baryon asymmetry as a function of parameters separated into three categories: those we know now, those we hope to know, and those we will never know. It then seems straightforward to study how the asymmetry depends on, for instance, θ_{13}. But in practise it is anything but transparent (see eqn 11): the asymmetry is independent of U in the Casas-Ibarra parametrisation, but does depends on U in the bottom-up version. That is, the choice of parametrisation for

the unmeasurables, changes the dependence of one observable (the baryon asymmetry) on another (θ_{13}). It would be better to ask "is ϵ_1 *sensitive* to θ_{13}?"—this has a unique and useful answer, as discussed in the next section.

The aim of this section is to explore how different coordinates on seesaw parameter space depend on each other, and what we mean by "depends on" and "independent" . I start by reviewing some contradictory statements which can be derived using various parametrisations. Then I present a toy model using parametrisations of the plane, where these same contradictions arise, and where the resolution is obvious. Lastly I suggest how the analogy of the plane could be related to the seesaw.

It has been claimed in various papers that ϵ_1, the CP asymmetry of thermal leptogenesis, is independent of the leptonic mixing matrix U. This seems intuitively reasonable, because leptogenesis involves the ν_R, and is independent of Y_e. In the limit of hierarchical ν_R:

$$\epsilon_1 \simeq -\frac{3M_1}{8\pi[Y_\nu Y_\nu^\dagger]_{11}} \Im\{Y_\nu \kappa^* Y^T\} = -\frac{3M_1}{8\pi} \frac{\Im\{R_{1j}^2 \kappa_j^2\}}{|R_{1k}|^2 \kappa_k} \tag{10}$$

$$\propto \Im\left\{ V_L U D_\kappa^3 U^T V_L^T D_{Y_\nu}^{-2} V_L^* U^* D_\kappa U^\dagger V_L^\dagger D_{Y_\nu}^{-2} \right\} \tag{11}$$

where the second equality of (10) is in the Casas-Ibarra parametrisation. To translate eqn (10) into bottom-up coordinates, requires calculating the mass and eigenvector (first colomn of V_R) of ν_{R1}, which gives short analytic formulae in some limits. However, for hierarchical ν_R (the limit in which eqn (10) is valid), ϵ_1 is proportional to a Jarlskog invariant[16], which gives eqn (11). We see that U does not appear in the expression for ϵ_1 in Casas-Ibarra, but does appear in the bottom-up parametrisation. So it is unclear whether ϵ_1 depends on U—what do we mean by "depend"? If a mathematical definition can be constructed, then there should be a unique answer.

We can draw an analogy between coordinate choices on a manifold, and parametrisation choices for the seesaw. Different coordinate choices on the plane, all used with the same [a] metric $\delta_{\alpha\beta}$, give confusing results that ressemble the puzzle about whether ϵ depends on θ_{13}. If we use the appropriate metrics, results are independent of the coordinate system, which can be chosen for calculational convenience. There is no metric given on "seesaw parameter space", but this analogy suggests that inventing one would resolve the confusion.

[a] Of course, we know that this is wrong; the metric should change with the coordinate system.

Consider two choices of coordinates on the upper half plane:

(1) the cartesian (y, z) with $y > 0$ and metric $g_{\alpha\beta} = I$.
(2) $R = \sqrt{y^2 + z^2}$ and $Z = z$, with $R > Z$ and metric

$$g_{AB} = \frac{R^2}{R^2 - Z^2} \begin{bmatrix} 1 & -Z/R \\ -Z/R & 1 \end{bmatrix} \tag{12}$$

These are equally good coordinate choices for the same flat 2-d surface. The seesaw analogy we want to address is: does R "depend" on Z?

In any coordinate system, the coordinates vary independently. So by definition

$$\frac{\partial R}{\partial Z} = 0 \tag{13}$$

which could be taken to mean that " R is independent of Z". A more intuitive quantity is the total derivative, or by analogy with general relativity, the change of R, treated as a scalar function, along the curve of varying Z:

$$\begin{pmatrix} \frac{\partial R}{\partial y} & \frac{\partial R}{\partial z} \end{pmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{pmatrix} \frac{\partial Z}{\partial y} \\ \frac{\partial Z}{\partial z} \end{pmatrix} = \frac{Z}{R} \tag{14}$$

which is the expected answer. Notice that we need to know how to transform to cartesian coordinates (equivalently, the metric on R, Z space) for this calculation.

To summarise, ϵ and θ_{13} are functions of seesaw parameter space, and can be defined such that

$$\frac{\partial \epsilon}{\partial \theta_{13}} = 0 \tag{15}$$

by a suitable choice of parameters. However, a better measure of whether ϵ depends on θ_{13} would be something like eqn (14). To evaluate this, we need a metric on seesaw parameter space [b].

How to choose this metric? The top-down parametrisation is the most natural, so in two generations, the obvious choice is to take $\{D_{Y_e}, V_L, D_{Y_\nu}, V_R, D_M\}$ as cartesian coordinates. With this metric, it is straightforward to show that ϵ does vary with the angle of the matrix U. (This is simple, because in 2 generations it is easy to calculate the angle of W_L in terms of RH parameters.) However, in three generations, "distance"

[b] When doing seesaw parameter space scans, one must choose the distribution of input points in parameter space. This number density ("measure on parameter space") is motivated by some theoretical model for the origin of seesaw parameters, so is not intrinsic to the seesaw. Therefore it is not related to this "metric".

on the unitary transformations should be invariant under reparametrizations ($e.g. V_R = U_{12}U_{13}U_{23}$ or $= U_{23}U_{13}U_{12}$), suggesting a metric similar to the one for polar coordinates.

It *is* clear, from this section, that the "dependance" of one seesaw observable on another in *not* clear. For example, the coordinates on seesaw parameter space can be chosen such that either ϵ_1 is a function of the MNS matrix, or it is not. This confusion can be resolved by inventing a notion of "orthogonality" for coordinates, that is, a metric on parameter space. However, the metric seems an esoteric solution, and how to find the correct one is not obvious.

5. Rethink: what happens in the Standard Model?

In the Standard Model, the Lagrangian parameters can be reconstructed from data—in fact, there are many more measurements than parameters, so the SM is tested at part-per-mil accuracy. But some parameters are better determined than others, so the difference with respect to the seesaw is just the size of the error bars.

In the SM, the key is the *sensitivity* of data to a parameter. For instance, to determine m_t from electroweak data, one should choose an observable with large m_t^2 corrections, and a parametrisation (*eg*, definition of s_W^2), where these are easy to identify. If the parameters other than m_t are sufficiently well determined, a range for m_t can be extracted [c]. This is self-evident; the data allows a model to occupy a subset (often a multi-dimensional ellipse) in parameter space.

We say an observable Ob is sensitive to a parameter P, if measuring Ob constrains P to sit in a certain range. Conversely, Ob is insensitive to P, if measuring Ob is consistent with any value of P (possibly because one ajusts other unknowns to compensate for variations in P).

So returning to the seesaw, one could conclude that "does ϵ_1 depend on θ_{13}?" is the wrong question. If instead, one asks "is ϵ_1 *sensitive* to θ_{13}?", then the answer at present is clearly no. It is easy to see, in the parametrisation using R, that any value of θ_{13} is consistent with the observed baryon a symmetry.

[c] In reality this is a crude approx to doing a combined fit.

6. Summary

The seesaw generates small neutrino masses, by introducing heavy majorana ν_Rs, which share a Yukawa coupling with the lepton doublets of the Standard Model. It is theoretically possible to establish a 1-1 correspondance between observables (in the quantum mechanical sense), and the 21 parameters of the seesaw (type I, 3 generations). This correspondance, and two other parametrisations of the seesaw, are discussed in section 2. Unfortunately, this peculiarity of the seesaw does not mean the parameters can be extracted from data; some of the "observables" are not realistically measurable, and others cannot be determined accurately enough (see section 3). This makes the seesaw mechanism difficult to test, according to the definition of test outlined in the introduction.

This is sad because the dream test of the seesaw would be to extract its parameters from data, calculate the baryon asymmetry produced in leptogenesis—and get the right answer. More realistically, we can ask "is the baryon asymmetry *sensitive* to any of the seesaw's measurable parameters?" For instance, does generating the baryon asymmetry by a specific leptogenesis scenario imply that θ_{13}, or the phase δ, should occupy restricted ranges? Again, the answer sadly seems to be "no". More generally, one could study which observables are sensitive to which parameters, *e.g.* would $BR(\mu \to e\gamma) \neq 0$ restrict the majorana phases of MNS [d]? Most studies to date have looked at whether an observable O "depends" on a parameter P—which is not such a useful question, because the answer depends on the parametrisation. Section 4 attempts to construct a parametrisation-independent definition of "depend", not very successfully. So it is better to ask if O is *sensitive* to P, which does have a unique answer, as discussed in section 5.

Acknowledgements

I wish the seesaw many happy returns, and thank the organisers for a very enjoyable birthday celebration, with many interesting speakers and participants. In particular, I thank S Vempati for clarifying questions, S Lavignac for discussions and a careful reading of the manuscript, and S Petcov for many interesting discussions about seesaw parametrisations.

[d]In the Casas-Ibarra parametrisation, $BR(\mu \to e\gamma)$ depends on theses phases [18]

References

1. M. Gell-Mann, P. Ramond and R. Slansky, *Proceedings of the Supergravity Stony Brook Workshop*, New York 1979, eds. P. Van Nieuwenhuizen and D. Freedman; T. Yanagida, *Proceedinds of the Workshop on Unified Theories and Baryon Number in the Universe*, Tsukuba, Japan 1979, ed.s A. Sawada and A. Sugamoto; R. N. Mohapatra, G. Senjanovic, *Phys.Rev.Lett.* **44** (1980)912.
2. M. Fukugita and T. Yanagida, Phys. Lett. B **174** (1986) 45.
3. *see e.g.* W. Buchmüller and M. Plümacher, Int. J. Mod. Phys. A **15** (2000) 5047 [arXiv:hep-ph/0007176], *or* G. F. Giudice, A. Notari, M. Raidal, A. Riotto and A. Strumia, Nucl. Phys. B **685** (2004) 89 [arXiv:hep-ph/0310123].
4. *a list of other possibilities can be found in, e.g.,* L. Boubekeur, S. Davidson, M. Peloso and L. Sorbo, Phys. Rev. D **67** (2003) 043515 [arXiv:hep-ph/0209256].
5. G. C. Branco, L. Lavoura and M. N. Rebelo, Phys. Lett. B **180** (1986) 264. A. Santamaria, Phys. Lett. B **305** (1993) 90 [arXiv:hep-ph/9302301].
6. S. Davidson and A. Ibarra, JHEP **0109** (2001) 013 [arXiv:hep-ph/0104076].
7. J. A. Casas and A. Ibarra, Nucl. Phys. B **618** (2001) 171 [arXiv:hep-ph/0103065].
8. V. Barger, S. L. Glashow, P. Langacker and D. Marfatia, Phys. Lett. B **540** (2002) 247 [arXiv:hep-ph/0205290].
9. F. Borzumati and A. Masiero, Phys. Rev. Lett. **57** (1986) 961.
10. M. Ibe, R. Kitano, H. Murayama and T. Yanagida, arXiv:hep-ph/0403198.
11. N. Arkani-Hamed, H. C. Cheng, J. L. Feng and L. J. Hall, Phys. Rev. Lett. **77** (1996) 1937 [arXiv:hep-ph/9603431].
12. *see, e.g.* [7], *or* J. Hisano and D. Nomura, Phys. Rev. D **59** (1999) 116005 [arXiv:hep-ph/9810479]. S. Lavignac, I. Masina and C. A. Savoy, Nucl. Phys. B **633** (2002) 139 [arXiv:hep-ph/0202086].
13. N. Arkani-Hamed, J. L. Feng, L. J. Hall and H. C. Cheng, Nucl. Phys. B **505** (1997) 3 [arXiv:hep-ph/9704205].
14. Y. Grossman and H. E. Haber, Phys. Rev. Lett. **78** (1997) 3438 [arXiv:hep-ph/9702421].
15. A. Broncano, M. B. Gavela and E. Jenkins, Phys. Lett. B **552** (2003) 177 [arXiv:hep-ph/0210271].
16. S. Davidson and R. Kitano, JHEP **0403** (2004) 020 [arXiv:hep-ph/0312007].
17. S. Davidson, JHEP **0303** (2003) 037 [arXiv:hep-ph/0302075].
18. S Petcov at ν'04.